

# Incentive Mechanism Design for Joint Resource Allocation in Blockchain-based Federated Learning

Zhilin Wang, Qin Hu, Ruinian Li, Minghui Xu, and Zehui Xiong

**Abstract**—Blockchain-based federated learning (BCFL) has recently gained tremendous attention because of its advantages, such as decentralization and privacy protection of raw data. However, there has been few studies focusing on the allocation of resources for the participated devices (i.e., clients) in the BCFL system. Especially, in the BCFL framework where the FL clients are also the blockchain miners, clients have to train the local models, broadcast the trained model updates to the blockchain network, and then perform mining to generate new blocks. Since each client has a limited amount of computing resources, the problem of allocating computing resources to training and mining needs to be carefully addressed. In this paper, we design an incentive mechanism to help the model owner (MO) (i.e., the BCFL task publisher) assign each client appropriate rewards for training and mining, and then the client will determine the amount of computing power to allocate for each subtask based on these rewards using the two-stage Stackelberg game. After analyzing the utilities of the MO and clients, we transform the game model into two optimization problems, which are sequentially solved to derive the optimal strategies for both the MO and clients. Further, considering the fact that local training related information of each client may not be known by others, we extend the game model with analytical solutions to the incomplete information scenario. Extensive experimental results demonstrate the validity of our proposed schemes.

**Index Terms**—Federated learning, blockchain, resource allocation, incentive mechanism, game theory



## 1 INTRODUCTION

SINCE its emergence in 2016, federated learning (FL) has been greatly developed and widely applied in many fields, such as Internet of Things [1]–[3], smart transportation [4], [5] and healthcare [6]–[8]. One of the most important advantages of FL is that there is no transmission of raw data from local devices (i.e., clients) to the centralized server for model training; instead, by training models on clients and aggregating all local models, FL significantly reduces the possibility of leaking data privacy to a large extent [9]. However, some challenges still may restrain the implementation of FL, e.g., the risk of the single point of failure, malicious attacks from participated clients, and the lack of participation incentives [10]–[13].

In recent years, researchers resort to blockchain technology to tackle the challenges of FL, where the blockchain system usually works as a decentralized system to provide incentives and data verification [14]–[18]. The combination of blockchain and FL is termed blockchain-based FL (BCFL). In the BCFL framework, model updates submitted by clients

will be verified by miners before the global aggregation algorithm is conducted. Once the global model is obtained, it will be updated into the main chain that all qualified participants can access. Though BCFL can partially address the aforementioned challenges of traditional FL, some remaining issues still need to be addressed.

One of the most critical problems in BCFL is the resource allocation of clients. Firstly, clients in the BCFL system are heterogeneous with different computational resources, and they usually have other tasks to complete while handling the BCFL task, so a universal resource allocation scheme for all clients is not practical. In addition, the whole system may not work effectively and sustainably if no reasonable rewards are allocated to clients. Furthermore, both training and mining in the framework of BCFL consume a significant amount of resources and time, and thus it is difficult for clients to appropriately allocate their limited resources to ensure the performance of the global model during the required time period. Lastly, since the system may not know the amount of training data each client owns, it can be challenging for the model owner (MO), i.e., the BCFL task publisher, to make proper decisions regarding the reward distribution.

There exist very few studies that tackle the above challenges [19], [20]. They are mainly based on two assumptions that are not practical: 1) all clients have identical computational power and data volume; and 2) the system knows all the information about the computation resources of clients. Besides, there are several studies focusing on resource allocation [21], [22] and incentive mechanism design [23], [24] in BCFL. But these schemes cannot address above mentioned challenges. To fill the gap, we propose an

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incentive mechanism for joint resource allocation on clients in BCFL that can be applied to the incomplete information scenario without two impractical assumptions.

For the first challenge regarding the unbalanced distribution of resources on clients, we let clients decide how much computational power they are willing to devote to the training and mining tasks. By this means, clients can flexibly allocate computation resources for their own tasks. In addition, training and mining are performed sequentially in our model, and the amount of computational power devoted to these two tasks can be different.

To overcome the second challenge of motivating clients to join BCFL, we design an incentive mechanism to reward clients. Training and mining are two different tasks that require a different amount of computational power; thus, the rewards should also be different. To ensure a fair distribution of rewards to all clients, we employ the approach of Shapley Value (SV) [25] to determine clients' contributions in the training process, which will affect the constraints in their respective optimization problems.

To address the last two challenges, we build the Stackelberg game model under the complete and incomplete information situations, which are solved separately but with different insights. Our system can make optimal decisions based on the derived optimal solutions in different information conditions.

In summary, our contributions can be summarized as below:

- In the BCFL system with heterogenous clients, we model the resource allocation problem as a two-stage Stackelberg game to help the MO make decisions on assigning how many rewards to each client for training and mining and to assist clients in determining the corresponding amount of computational power to be devoted in each subtask, via maximizing their respective utilities.
- In order to maintain the stability and sustainability of the whole BCFL system, we design a fair reward allocation scheme inspired by SV to calculate the rewards for clients based on their contributions to the training process.
- Considering that the training related information of devices may not be known to others in the practical application scenario, we further study the resource allocation mechanism under the incomplete information situation and derive the optimal solutions accordingly.
- We test our proposed resource allocation mechanisms through extensive experiments. The experimental results show that these mechanisms are effective.

The rest of this paper is organized as follows. We introduce the system model and problem formulation based on the two-stage Stackelberg game in Section 2. The detailed models and solutions under complete and incomplete information scenarios are reported in Section 3 and Section 4, respectively. Experimental evaluations are presented in Section 5. We present the related work in Section 6. Finally, we conclude this paper in Section 7.

## 2 SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we will illustrate the system model of our considered blockchain-based federated learning (BCFL) and then formulate the problem from the perspective of resource allocation and incentive mechanism design based on the Stackelberg game. For convenience, we list the key notations in Table 1.

TABLE 1: Key Notations.

Notation	Meaning
$\mathcal{N}$	The set of clients
$N$	The total number of clients
$q_i$	The maximum number of client $i$ 's CPU cycle per second
$qt_i$	The number of CPU cycles per second used to train
$q_{mi}$	The number of CPU cycles per second used to mine
$pt_i$	The unit price for training to client $i$
$p_{mi}$	The unit price for mining to client $i$
$\pi$	The number of training iterations for clients during one round of BCFL to submit model update
$D_i$	The data size of client $i$
$d_i$	The number of CPU cycles used for training each data sample
$\mu_i$	The total CPU cycles required to finish the local training for generating model updates
$T_i$	The time spent on training for client $i$
$\psi$	The total CPU cycles used to mine for each client
$T_{mi}$	The time spent on mining for client $i$
$U_i$	The utility of client $i$ in one round of BCFL
$U_{mo}$	The utility of the MO in one round of BCFL
$q_{t_i}^*$	The optimal CPU cycles per second for training
$q_{mi}^*$	The optimal CPU cycles per second for mining
$p_{t_i}^*$	The optimal unit price for training to client $i$
$p_{mi}^*$	The optimal unit price for mining to client $i$

### 2.1 System Overview

Inspired by [26], we consider one of the most widely used BCFL frameworks, i.e., the fully coupled BCFL system, where each client of FL also works as the blockchain node and thus has to handle both FL-related and blockchain-related computing activities. Without loss of generality, we term the local devices as clients and call the FL-related computing activities *training* and blockchain-related computing activities *mining*. The topology of the BCFL system is shown in Fig. 1, which is consisted of multiple clients and one blockchain system. The set of clients can be denoted as  $\mathcal{N} = \{1, \dots, i, \dots, N\}$  with  $N$  representing the total number of clients in the BCFL system. The task requester, i.e., the model owner (MO), can access the BCFL system and publish tasks, aiming to receive a well-trained final global model from the BCFL system. After the FL task is published on the blockchain, clients train their local models and broadcast the obtained model updates to the blockchain network once the local training process is finished.

It is worth noting that the BCFL system considered in this paper can adopt any blockchain consensus depending on the application scenario. Specifically, Proof of Work (PoW) [27] can be employed as the consensus protocol to provide the security guarantee, while other lightweight consensus, such as Practical Byzantine Fault Tolerance (PBFT) [28], Delegated Proof of Stake (DPoS) [29], and Raft [30], can also be implemented to reduce the resource consumption. There will be a generator of new block after running

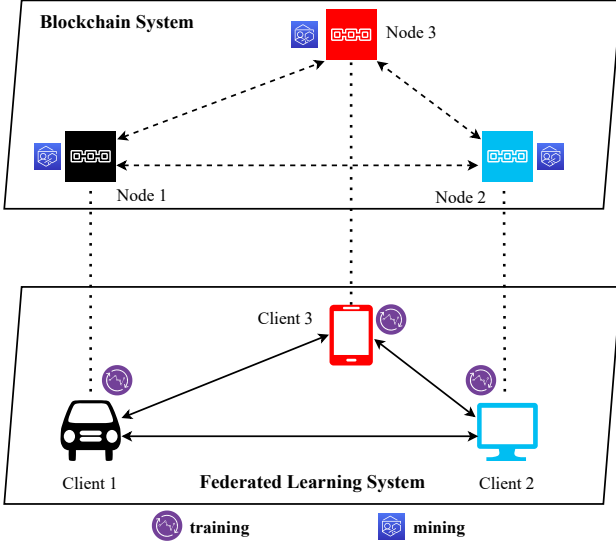


Fig. 1: An illustration of the BCFL system, which is the fully coupled BCFL with the FL clients also being the blockchain nodes. First, clients train local models and submit their model updates to the blockchain network; then, clients start mining to determine a block generator, which will also be responsible for global model aggregation.

blockchain consensus in the BCFL system, which will also be responsible for aggregating the local models to derive the global model. In addition, the considered BCFL system is compatible with any type of blockchain; in other words, the BCFL system can also apply public blockchain and private blockchain in addition to the consortium blockchain.

We can describe the workflow of our BCFL system after the MO publishes the task as below: 1) once receiving the FL task, each client trains the local model and then broadcasts the model updates to the blockchain network; 2) clients run the consensus protocol and determine a new block generator who will aggregate the received model updates to derive a new global model; 3) then the block generator will create a new block, containing all the submitted updates and the new global model.

In our considered BCFL system, each client would be responsible for training and mining, which is defined as the BCFL task. These two procedures are not parallel; in other words, mining can only start after the training is completed. In practice, clients may need to handle other tasks besides the BCFL task. Given that their computational resources are limited, they have to allocate available resources to both training and mining carefully. Besides, to motivate clients to complete the BCFL task, the MO usually provides a certain amount of rewards; however, since the MO's budget is limited, the distribution of rewards to clients becomes challenging to obtain a high-performance global model in an efficient manner.

## 2.2 Utility Models

Based on the above analysis, we design an incentive mechanism to assist the resource allocation for our proposed BCFL system, so that the MO can distribute rewards to clients properly and get a well-trained global model, and

clients can allocate the computing resources (i.e., CPU cycles per second) to training and mining reasonably and gain satisfying rewards. In this part, we build the utility models of both the MO and clients from resource allocation and incentive mechanism perspectives.

### 2.2.1 Client's Utility

We assume that the maximum number of client  $i$ 's CPU cycles per second is  $q_i$ , and the number of CPU cycles per second used to train and mine are  $q_{ti}$  and  $q_{mi}$ , respectively. Then we have  $q_{ti}, q_{mi} \leq q_i$ . Let  $\pi$  be the number of training iterations for clients during one round of BCFL to submit a model update, which is usually fixed for all clients. Let  $D_i$  be the number of the data size of client  $i$ , and  $d_i$  be the number of CPU cycles used for training each data sample. Therefore, we can define the total CPU cycles required to finish the local training to generate model updates as:

$$\mu_i = \pi d_i D_i.$$

Since any client  $i$  can decide its CPU cycles used to train the local model, the time used to finish the local training varies for each client. We can calculate the time spent on training for client  $i$  via  $T_{ti} = \frac{\mu_i}{q_{ti}}$ . Besides, we denote the total CPU cycles used to mine for each client as  $\psi$ , which is the same for all clients since mining a new block in the blockchain system usually consumes fixed computational resources. Thus, the time spent on mining can be calculated as:

$$T_{mi} = \frac{\psi}{q_{mi}}.$$

So we can have the total time cost of client  $i$  to finish a round of BCFL task as  $T_i = T_{ti} + T_{mi}$ . Since it is impossible to let  $T_{ti}$  and  $T_{mi}$  be limitless according to the convergence time requirement, we denote the upper bound of time consumption in one round of BCFL by  $T$ . Thus, we have  $T_i \leq T$ .

In order to encourage clients to join BCFL, the MO provides some rewards to clients, where the prices per second for training and mining are denoted as  $p_{ti}$  and  $p_{mi}$ , respectively. Clients can allocate unit CPU cycles for training and mining based on the unit prices given by the MO. Then the rewards of client  $i$  for training and mining to generate one round of local model updates are calculated by

$$\begin{aligned} R_{ti} &= T_{ti} p_{ti}, \\ R_{mi} &= T_{mi} p_{mi}. \end{aligned}$$

Thus, the total rewards for client  $i$  in one round of BCFL is  $R_i = R_{ti} + R_{mi}$ .

Based on a widely used model in [31] for calculating the computational energy consumption, we can respectively calculate the energy costs for training and mining as

$$\begin{aligned} C_{ti} &= \rho_i \mu_i q_{ti}^2, \\ C_{mi} &= \rho_i \psi q_{mi}^2, \end{aligned}$$

where  $\rho_i$  is the parameter correlated to the chip architecture. In this way, the total cost<sup>1</sup> can be calculated as  $C_i = C_{ti} + C_{mi}$ .

1. As for the communication cost, since the sizes of the clients' submissions are the same, we can consider it as a constant value, which cannot be optimized anymore and thus is omitted here.

Finally, we can obtain the utility of client  $i$  in one round of BCFL as

$$\begin{aligned} U_i &= R_i - C_i \\ &= \frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi} - \rho_i \mu_i q_{ti}^2 - \rho_i \psi q_{mi}^2. \end{aligned} \quad (1)$$

## 2.2.2 MO's Utility

The main concerns related to the utility of the MO are the performance of the global model, the time consumption, and the rewards paid to all participants in each round of BCFL, where the first one is a sort of revenue and the last two are related to the cost for the MO.

Generally, the performance of an ML model will be affected by the number of CPU cycles spent on training. Thus, we define the performance of the global model after one round of local training and mining as  $G$ , which can be calculated by  $G = f(\sum_{i=1}^N \mu_i)$ . Here  $f(\cdot)$  is a monotonically increasing function, indicating that the more CPU cycles used for the local training by all clients, the better performance of the global model after aggregation will be achieved. As for the MO, its utility depends on the performance of the BCFL system ( $G$ ), total time cost ( $\sum_{i=1}^N (T_i)$ ), and total rewards distributed to clients ( $\sum_{i=1}^N (R_i)$ ). Thus, the utility of the MO in one round of BCFL can be expressed as

$$\begin{aligned} U_{mo} &= f\left(\sum_{i=1}^N \mu_i\right) - \xi \sum_{i=1}^N (T_i + R_i) \\ &= f\left(\sum_{i=1}^N \mu_i\right) - \xi \sum_{i=1}^N \left(\frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} + \frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi}\right), \end{aligned} \quad (2)$$

where  $\xi > 0$  is a scalar parameter to balance the revenue and cost.

## 2.3 Problem Formulation using Two-stage Stackelberg Game

According to the above analysis of our system model, client  $i$  provides its computational power to finish BCFL tasks based on the rewards given by the MO. In other words, the unit prices  $p_{ti}$  and  $p_{mi}$  determine the unit computational power  $q_{ti}$  and  $q_{mi}$ . We can formulate the interactions between clients and the MO as a two-stage Stackelberg game, which is widely used for the complete information dynamic game [32]. In this game, the MO determines the unit prices of the CPU frequency used for training and mining, and then client  $i$  decides its CPU cycles per second based on the received prices, which means that the decision of client  $i$  is impacted by the decision of the MO. In this case, we can define the process of the two-stage Stackelberg game as below:

- Stage I: The MO sets the unit prices per second for training and learning for each client, i.e.,  $p_{ti}$  and  $p_{mi}$ , via maximizing its own utility, which is specifically based on its budget and the total number of CPU cycles consumed for training submitted by each client. Considering the distribution's fairness in setting prices, we need to design a fair reward allocation scheme here.

- Stage II: After receiving the unit prices from the MO, clients determine their corresponding computational power, i.e.,  $q_{ti}$  and  $q_{mi}$ , through optimizing their respective utilities.

In practical situations,  $q_{ti}$  and  $q_{mi}$  are not independent of each other because of time and reward budget constraints; similarly,  $p_{ti}$  and  $p_{mi}$  influence each other as well. Therefore, we should consider these constraints when modeling to make the decisions reasonable.

Recall  $\mu_i$  in Section 2.2.1, and we know that  $\mu_i$  is a variable correlated to the data size of client  $i$  and the performance of the corresponding device, which may not always be known to the MO or the system. As for  $\psi$ , it can be predefined by the system since generating a new block usually consumes a constant amount of resources. Therefore, we can classify the two-stage Stackelberg game into information complete and incomplete scenarios based on whether  $\mu_i$  is known to the MO. The models derived for these two scenarios are different, and hence the strategies of the MO and clients are different accordingly, which will be explored in Sections 3 and 4.

## 3 RESOURCE ALLOCATION WITH COMPLETE INFORMATION

In this section, we will elaborate on the expressions of the proposed Stackelberg game model and the corresponding solutions for clients and the MO in the scenario of complete information, which means that the MO makes its decisions when  $\mu_i$  of each client is known as a prior. First, we propose a fair reward allocation scheme for clients, and then we transfer the two-stage Stackelberg game into two separate optimization problems that are resolved sequentially. The methodology we adopt to solve the two problems is backward induction, which requires analyzing the optimal strategies of Stage II first and then the strategies of Stage I.

### 3.1 Fair Reward Allocation

Before we formulate the game model, we should clarify the fair reward allocation scheme first. In our system, we consider that each client has an equal chance to participate in both the training and mining processes with fair rewards. And since the allocation of rewards to each client in training and mining significantly impacts the system fairness and further participation willingness, we need to design a fair reward allocation scheme. Although we have already defined the payoff of each client during the training and mining processes in the above section, it is necessary to investigate their upper bounds based on the MO's rewards budget. And the rewards distribution should not only be associated with the device's computing power but also take into account the performance of its work. On the one hand, the reward budget of the MO and the rewards that each client can get is limited; on the other hand, if the resources are allocated only based on the computing power devoted, it could lead to a situation where devices with sufficient computing power take most of the rewards, while devices with less power cannot get enough rewards, making the system unstable and unsustainable.

### 3.1.1 Upper Bound of Rewards for Mining

For simplicity, we set a fixed total reward budget  $\eta$  in each round of BCFL. Since the computational power consumed by generating a new block is constant, with  $\eta_m$  denoting the upper bound of the reward for mining that all clients can receive, we have:

$$\frac{\psi}{q_{mi}} p_{mi} \leq \bar{R}_{mi} = \frac{\eta_m}{N},$$

where  $\bar{R}_{mi}$  is the upper bound of the reward for mining that each client can get.

### 3.1.2 Upper Bound of Rewards for Training

Since the devices in our BCFL system are assumed to be heterogeneous and may have different computing capabilities, we cannot simply distribute the rewards evenly to each client. To guarantee the fairness of reward distribution, we allocate rewards based on the contribution of each client in the training process. Considering that Shapley Value (SV) [25] is a methodology that can distribute the rewards to participants according to their respective contributions, here we apply it to facilitate reward distribution. The SV of client  $i$  is defined as

$$SV_i(\mathcal{N}, v) = \sum_{i \notin S, S \subseteq \mathcal{N}} \frac{s!(N-s-1)!}{N!} (v(S \cup i) - v(S)), \quad (3)$$

where  $S \subseteq \mathcal{N}$  is a subset of clients and  $s = |S|$  is the number of devices in the set  $S$ ;  $v(S)$  is a function describing the performance of the training result with the client set  $S$ . Then, we give the expression of function  $v(S)$ . Recall  $G$  in Section 2.2.2, we can assume that  $v(S)$  is a function correlated to  $G$  and it can be defined as

$$v(S) = W - \left\| \frac{\sum_{i=1}^s G}{s} - g \right\|_2, \quad (4)$$

where  $W = \max_{S \subseteq \mathcal{N}} \left\| \frac{\sum_{i=1}^s G}{s} - g \right\|_2$  and  $\|\cdot\|_2$  is the Euclidean norm;  $g$  is the targeted performance value. Then, we can calculate the upper bound of the reward distributed to each device for training as:

$$\bar{R}_{ti} = \frac{SV_i(\mathcal{N}, v)}{v(\mathcal{N})} (\eta - \eta_m).$$

For each client, its rewards should not exceed the upper bound so that we can have the following constraint:

$$\frac{\mu_i}{q_{ti}} p_{ti} \leq \bar{R}_{ti}.$$

It is worth pointing out that with a fixed reward budget of the MO, the maximum rewards per client will decrease as the number of clients increases.

### 3.2 Stage II: Clients Set CPU Cycles Per Second based on Unit Rewards

Since each client  $i$  has a limited amount of computational resources and should follow the working rules of BCFL, the goal of client  $i$  is to maximize its utility as follows:

$$\text{Problem 1: } \max : U_i, \quad \text{s.t. } \frac{\mu_i}{q_{ti}} p_{ti} - \rho_i \mu_i q_{ti}^2 \geq 0, \quad (5)$$

$$\frac{\psi}{q_{mi}} p_{mi} - \rho_i \psi q_{mi}^2 \geq 0, \quad (6)$$

$$\frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} \leq T, \forall i \in \mathcal{N}, \quad (7)$$

where the first two constraints (5) and (6) mean that client  $i$  wishes to gain non-negative payoffs in both training and mining; and the last constraint (7) indicates that client  $i$  should finish the working process, including training and mining, within the time period  $T$ .

It is clear that **Problem 1** is a nonlinear optimization problem with inequality constraints, so we adopt the method of Karush-Kuhn-Tucker (KKT) conditions to solve it. By solving **Problem 1**, we get the following theorem:

**Theorem 3.1.** *The optimal strategies of client  $i$  in the scenario of complete information are given by*

$$q_{ti}^* = \left( \frac{p_{ti}}{\rho_i} \right)^{\frac{1}{3}}, \quad (8)$$

$$q_{mi}^* = \frac{\psi}{T - \mu_i \left( \frac{\rho_i}{p_{ti}} \right)^{\frac{1}{3}}}. \quad (9)$$

*Proof.* The Lagrangian correlated to **Problem 1** is expressed as:

$$\begin{aligned} \mathcal{L}_1 = & \frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi} - \rho_i \mu_i q_{ti}^2 - \rho_i \psi q_{mi}^2 \\ & - \lambda_1 \left( \rho_i \mu_i q_{ti}^2 - \frac{\mu_i}{q_{ti}} p_{ti} \right) - \lambda_2 \left( \rho_i \psi q_{mi}^2 - \frac{\psi}{q_{mi}} p_{mi} \right) \\ & - \lambda_3 \left( \frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T \right), \forall i, \end{aligned}$$

where  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  are non-negative parameters correlated to the constraints of **Problem 1**.

The KKT conditions are as below:

$$\frac{\partial \mathcal{L}_1}{\partial q_{ti}} = \frac{\partial \mathcal{L}_1}{\partial q_{mi}} = 0, \forall i, \quad (10)$$

$$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \forall i, \quad (11)$$

$$\lambda_1 \left( \rho_i \mu_i q_{ti}^2 - \frac{\mu_i}{q_{ti}} p_{ti} \right) = 0, \forall i,$$

$$\lambda_2 \left( \rho_i \psi q_{mi}^2 - \frac{\psi}{q_{mi}} p_{mi} \right) = 0, \forall i,$$

$$\lambda_3 \left( \frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T \right) = 0, \forall i,$$

$$\frac{\mu_i}{q_{ti}} p_{ti} - \rho_i \mu_i q_{ti}^2 \geq 0, \forall i,$$

$$\frac{\psi}{q_{mi}} p_{mi} - \rho_i \psi q_{mi}^2 \geq 0, \forall i,$$

$$\frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} \leq T, \forall i. \quad (12)$$

According to (10), we can have

$$\frac{\partial \mathcal{L}_1}{\partial q_{ti}} = \frac{u_i \lambda_3}{q_{ti}^2} - 2(1 + \lambda_1) \rho_i u_i q_{ti}, \forall i. \quad (13)$$

Let the above equation equal to 0 and we have

$$\frac{\lambda_3}{2} = 2(1 + \lambda_1) \rho_i q_{ti}, \forall i. \quad (14)$$

Similarly, we have

$$\frac{\lambda_3}{2} = 2(1 + \lambda_2) \rho_i q_{mi}, \forall i. \quad (15)$$

Then, let's consider equation (12). Assume that  $\frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T \neq 0, \forall i$ , according to (12), we have  $\lambda_3 = 0$ . From (14), we can see that if  $\lambda_3 = 0$ , this equation will be  $2(1 + \lambda_1) \rho_i q_{ti} = 0, \forall i$ , then we have  $\lambda_1 = -1 < 0$ . Since  $\lambda_1$  is constrained by (11), it should always be non-negative. Therefore, this assumption is invalid. We can obtain the same conclusion from (15) as well. So we can conclude that for any  $i$ , equation  $\frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T = 0$  is always satisfied. In this way,  $\lambda_3 > 0$  can be deduced.

Based on the KKT conditions and  $\frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T \neq 0, \forall i$ , we can analyze the optimal solutions of **Problem 1** as follows:

**Case 1:**  $\lambda_1 = \lambda_2 = 0, \frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T = 0, \forall i$ .

In this case, since  $\lambda_1 = \lambda_2 = 0$ , we can derive  $q_{ti} = q_{mi} = \left(\frac{\lambda_3}{2\rho_i}\right)^{\frac{1}{3}} \geq 0$  using (14) and (15), respectively. But  $\lambda_3$  is a non-negative parameter, and it is not a constant value, so we still can not get the optimal solutions of **Problem 1**. Thus, this case is not suitable.

**Case 2:**  $\rho_i \mu_i q_{ti}^2 - \frac{\mu_i}{q_{ti}} p_{ti} = \rho_i \psi q_{mi}^2 - \frac{\psi}{q_{mi}} p_{mi} = 0, \forall i, \frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T = 0, \forall i$ .

By solving  $\rho_i \mu_i q_{ti}^2 - \frac{\mu_i}{q_{ti}} p_{ti} = 0, \forall i$  and  $\rho_i \psi q_{mi}^2 - \frac{\psi}{q_{mi}} p_{mi} = 0, \forall i$ , we have  $q_{ti} = \left(\frac{p_{ti}}{\rho_i}\right)^{\frac{1}{3}}, \forall i$  and  $q_{mi} = \left(\frac{p_{mi}}{\rho_i}\right)^{\frac{1}{3}}, \forall i$ .

Since  $\frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T = 0, \forall i$ , even though the above two functions can give the expression of the solution of **Problem 1**, it is still constrained by this function. In other words, one of the KKT conditions, i.e., (12), is not satisfied. Thus, this case is not suitable.

**Case 3:**  $\rho_i \mu_i q_{ti}^2 - \frac{\mu_i}{q_{ti}} p_{ti} = 0, \lambda_2 = 0, \frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T = 0, \forall i$ .

From (12), we can get the relationship between  $q_{ti}$  and  $q_{mi}$  is  $q_{mi} = \frac{\psi}{T - \frac{\mu_i}{q_{ti}}}$ . Solving  $\rho_i \mu_i q_{ti}^2 - \frac{\mu_i}{q_{ti}} p_{ti} = 0$  yields  $q_{ti} = \left(\frac{p_{ti}}{\rho_i}\right)^{\frac{1}{3}}, \forall i$ . Based on  $q_{mi} = \frac{\psi}{T - \frac{\mu_i}{q_{ti}}}$ , we let  $q_{ti} = \left(\frac{p_{ti}}{\rho_i}\right)^{\frac{1}{3}}, \forall i$ , then we can derive that  $q_{mi}(t) = \frac{\psi}{T - \frac{\mu_i}{\left(\frac{p_{ti}}{\rho_i}\right)^{\frac{1}{3}}}}, \forall i$ . From (14)

and (15), we have  $\lambda_1 = \frac{\lambda_3}{2\rho_i q_{ti}^3} - 1$  and  $\lambda_2 = \frac{\lambda_3}{2\rho_i q_{mi}^3} - 1$ . Since  $\lambda_3, q_{ti}, q_{mi}$  and  $\rho_i$  are positive, so  $\lambda_1 = \frac{\lambda_3}{2\rho_i q_{ti}^3} > 0$ , and  $\lambda_3$  can be large enough to make sure  $\lambda_1 = \frac{\lambda_3}{2\rho_i q_{ti}^3} \geq 1$ , thus  $\lambda_1 \geq 0$  can be guaranteed. Similarly,  $\lambda_2 \geq 0$  can be derived. From the above analysis, **Case 3** satisfies all the KKT conditions; therefore, the optimal solutions are obtained.

**Case 4:**  $\rho_i \psi q_{mi}^2 - \frac{\psi}{q_{mi}} p_{mi} = 0, \forall i, \lambda_1 = 0, \frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} - T = 0, \forall i$ . This case is similar to **Case 3**.

Based on the above analysis, the optimal solutions of

**Problem 1** are  $q_{ti}^* = \left(\frac{p_{ti}}{\rho_i}\right)^{\frac{1}{3}}, \forall i$ , and  $q_{mi}^* = \frac{\psi}{T - \mu_i \left(\frac{\rho_i}{p_{ti}}\right)^{\frac{1}{3}}}$ .

Thus **Theorem 3.1** is proved.  $\square$

From the above theorem, we can see that the number of optimal CPU cycles per second client  $i$  putting into training grows as the unit price for training given by the MO increases. The optimal CPU cycles per second devoted to mining are constrained by  $\psi$ , indicating that if the mining work requires more CPU cycles, client  $i$  should mine with a larger  $q_{mi}^*$ .

### 3.3 Stage I: MO Sets Unit Prices for Clients

The MO expects to get a global model with good performance consuming time and cost for rewards as less as possible, so its goal is to maximize the utility function  $U_{mo}$ , and the optimization problem can be formulated as follows:

$$\text{Problem 2: } \max : U_{mo}, \quad (16)$$

$$s.t. \quad \frac{\mu_i}{q_{ti}} p_{ti} \leq \bar{R}_{ti},$$

$$\frac{\psi}{q_{mi}} p_{mi} \leq \bar{R}_{mi}, \forall i \in \mathcal{N}, \quad (17)$$

where (16) and (17) are the constraints of individual rewards from training and mining to meet the MO's budget.

It is clear that **Problem 2** is also a nonlinear optimization problem, and  $U_{mo}$  is also concave, so we can list all the KKT conditions to find its maximum value. By solving **Problem 2**, we can have:

**Theorem 3.2.** *The optimal strategies of the MO in the scenario of complete information are:*

$$p_{ti}^* = \left(\frac{1}{\rho_i}\right)^{\frac{1}{2}} \left(\frac{\bar{R}_{ti}}{\mu_i}\right)^{\frac{3}{2}}, \quad (18)$$

$$p_{mi}^* = \frac{\bar{R}_{mi}}{T - (\rho_i \mu_i)^{\frac{3}{2}} \left(\frac{1}{\bar{R}_{ti}}\right)^{\frac{1}{2}}}. \quad (19)$$

*Proof.* The Lagrangian correlated to **Problem 2** is

$$\mathcal{L}_2 = f\left(\sum_{i=1}^N \mu_i\right) - \xi \sum_{i=1}^N \left(\frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} + \frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi}\right) - \theta_1 \left(\frac{\mu_i}{q_{ti}} p_{ti} - \bar{R}_{ti}\right) - \theta_2 \left(\frac{\psi}{q_{mi}} p_{mi} - \bar{R}_{mi}\right),$$

where  $\theta_1$  and  $\theta_2$  are the Lagrange multipliers correlated to the constraints of Problem 2. The following constraints should be met:

$$\frac{\partial \mathcal{L}_2}{\partial p_{ti}} = \frac{\partial \mathcal{L}_2}{\partial p_{mi}} = 0, \forall i,$$

$$\theta_1 \geq 0, \theta_2 \geq 0, \forall i,$$

$$\theta_1 \left(\frac{\mu_i}{q_{ti}} p_{ti} - \bar{R}_{ti}\right) = 0, \forall i,$$

$$\theta \left(\frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi} - \omega\right) = 0, \forall i,$$

$$\frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi} \leq \omega, \forall i.$$

$$\frac{\mu_i}{q_{ti}} p_{ti} \leq \bar{R}_{ti}, \forall i,$$

$$\frac{\psi}{q_{mi}} p_{mi} \leq \bar{R}_{mi}, \forall i.$$

First, let  $q_{ti} = q_{ti}^*$  and  $q_{mi} = q_{mi}(n)^*$ .

**Case 1:**  $\theta_1 = 0, \frac{\psi}{q_{mi}} p_{mi} - \bar{R}_{mi} = 0, \forall i.$

In this case, we can have

$$\frac{\partial \mathcal{L}_2}{\partial p_{ti}} = \frac{-(\mu_i(\xi p_{mi} + 2\xi p_{ti} + \theta_2 p_{mi}))}{3p_{ti}(\frac{p_{ti}}{\rho_i})^{\frac{1}{3}}}, \forall i.$$

Setting this equation equal to 0 yields  $p_{ti} = \frac{-p_{mi}(\xi + \theta_2)}{2\xi}$ . Obviously, we cannot find a positive  $\theta_2$  to satisfy this equation, making this solution invalid.

**Case 2:**  $\theta_2 = 0, \frac{\psi}{q_{ti}} p_{ti} - \bar{R}_{ti} = 0, \forall i.$

This case is similar to Case 1.

**Case 3:**  $\frac{\psi}{q_{ti}} p_{ti} - \bar{R}_{ti} = 0, \frac{\psi}{q_{mi}} p_{mi} - \bar{R}_{mi} = 0, \forall i.$

By solving  $\frac{\psi}{q_{ti}} p_{ti} - \bar{R}_{ti} = 0, \frac{\psi}{q_{mi}} p_{mi} - \bar{R}_{mi} = 0, \forall i$ , we can get (18) and (19). We can also prove that this case satisfies the rest of the KKT conditions.

Thus, **Theorem 3.2** is proved.  $\square$

In the optimal solutions above,  $p_{mi}^*$  and  $p_{ti}^*$  are highly correlated. This is because there are time and budget constraints so that  $p_{ti}^*$  and  $p_{mi}^*$  are not independent variables from each other. In other words, the MO needs to balance  $p_{ti}^*$  and  $p_{mi}^*$  to satisfy the constraints when making decisions. Furthermore, we can find that  $\mu_i$  and  $\psi$  influence the optimal decisions as well.

We summarize the resource allocation mechanism with complete information in Algorithm 1. The MO calculates the unit prices given to the client for training and mining first and then calculates its utility based on the previous unit prices (Lines 1-2). If  $U_{mo}$  is the optimal utility for the MO, then the optimal decisions of MO can be obtained (Lines 3-5). Next, the MO sends the unit prices to clients, and each client calculates the number of CPU cycles per second used for training and mining; if the utility for client  $i$  is optimal, client  $i$  can make its optimal decisions and start to train and mine (Lines 6-12). By observing Algorithm 1, we can see that its time complexity is mainly influenced by the number of clients, which can be expressed as  $\mathcal{O}(N)$ .

---

**Algorithm 1** Resource Allocation Mechanism with Complete Information

---

**Require:**  $T, \mu_i, \psi, \rho_i, \eta, \bar{R}_{mi}$

**Ensure:**  $q_{ti}^*, q_{mi}^*, p_{ti}^*, p_{mi}^*$

- 1: The MO calculates  $\hat{p}_{ti}$  and  $\hat{p}_{mi}$  via (18) and (19)
  - 2: The MO calculates  $U_{mo}$  based on  $\hat{p}_{ti}$  and  $\hat{p}_{mi}$  via (2)
  - 3: **if**  $U_{mo}(\hat{p}_{ti}, \hat{p}_{mi}) \geq U_{mo}(p_{ti}, p_{mi})$  **then**
  - 4:    $p_{ti}^* \leftarrow \hat{p}_{ti}, p_{mi}^* \leftarrow \hat{p}_{mi}$
  - 5: **end if**
  - 6: The MO sends  $p_{ti}^*$  and  $p_{mi}^*$  to the client  $i$
  - 7: **for**  $i \in \mathcal{N}$  **do**
  - 8:   Calculate  $\hat{q}_{ti}$  and  $\hat{q}_{mi}$  via (8) and (9)
  - 9:   **if**  $U_i(\hat{q}_{ti}, \hat{q}_{mi}) \geq U_i(q_{ti}, q_{mi})$  **then**
  - 10:      $q_{ti}^* \leftarrow \hat{q}_{ti}, q_{mi}^* \leftarrow \hat{q}_{mi}$
  - 11:     Client  $i$  uses  $q_{ti}^*$  to train and  $q_{mi}^*$  to mine
  - 12:   **end if**
  - 13: **end for**
  - 14: **return**  $q_{ti}^*, q_{mi}^*, p_{ti}^*, p_{mi}^*$
- 

In general, the case of complete information is an ideal situation, and we find that it mainly influences the optimal decisions of the MO. Therefore, we can study the optimal decisions in the case of incomplete information by adjusting the decision mechanism of the MO.

## 4 RESOURCE ALLOCATION WITH INCOMPLETE INFORMATION

In this section, we will discuss the game model in the case of incomplete information where the MO has no knowledge of the true value of  $\mu_i$  for each client. Thus, the MO needs to set the unit price so that each client has a non-negative payoff while ensuring that clients honestly report the value of  $\mu_i$ . Before designing the resource allocation mechanism, we first give two definitions below.

**Definition 4.1.** (*Individual Rationality*). The incentive mechanism for resource allocation is individually rational if the utility of client  $i$  given the rewards provided by the MO is non-negative, i.e.,

$$U_i(q_{ti}, q_{mi}, p_{ti}, p_{mi}, \mu_i) \geq 0, \forall i. \quad (20)$$

**Definition 4.2.** (*Incentive Compatibility*). The incentive mechanism for resource allocation is incentive compatible if each client can get the optimal utility by reporting its  $\mu_i$  truthfully, i.e.,

$$U_i(q_{ti}, q_{mi}, p_{ti}, p_{mi}, \mu_i) \geq U_i(q_{ti}, q_{mi}, p_{ti}, p_{mi}, \hat{\mu}_i), \forall i, \quad (21)$$

where  $\hat{\mu}_i$  represents any value of  $\mu_i$ .

Based on the previous analysis, clients make decisions based on their non-negative utility. Since clients should ensure that the rewards they receive are not less than the total costs they spend, they can participate in the BCFL task in such a situation. So in incomplete information, the MO needs to guarantee that its decisions should satisfy (20) to encourage clients to join the work. Besides,  $\mu_i$  of client  $i$  is not known by the MO, and the decisions of the MO are required to be based on the correct value of  $\mu_i$  reported by clients, so the MO needs to satisfy (21) when making the decisions.

Since the client sets the CPU cycles per second after the unit prices are given by the MO, the decisions of the client in the case of incomplete information are the same as those made under the complete information case as discussed in Section 3.2. Therefore, we will only focus on the derivation of the optimal strategies of the MO in this section.

With incomplete information, the MO has to ensure that the allocation of rewards to all clients is fair, the clients' utilities are non-negative, and clients report  $\mu_i$  truthfully. Thus, the decision-making problem of the MO with incomplete information can be transformed into the following optimization problem:

$$\begin{aligned} \text{Problem 3: } \max : & U_{mo} \\ \text{s.t. } & (16), (17), (20), (21), \\ & \forall i \in \mathcal{N}, \end{aligned}$$

where (20) and (21) are the constraints of individual rationality and incentive compatibility for the mechanism; (16) and (17) are the constraints of individual rewards for meeting the MO's budget.

To solve **Problem 3**, we can write it in Lagrangian form according to its optimization objective and constraints and then analyze its KKT conditions. The optimal solutions can be solved as below:

**Theorem 4.1.** *The optimal strategies of the MO in the scenario of incomplete information are*

$$p_{ti}^* = \left(\frac{1}{\rho_i}\right)^{\frac{1}{2}} \left(\frac{\bar{R}_{ti}}{\mu_i}\right)^{\frac{3}{2}}, \quad (22)$$

$$p_{mi}^* = \frac{\rho_i \psi^3}{\left(T - \mu_i \left(\frac{\rho_i}{p_{ti}^*}\right)^{\frac{1}{3}}\right)^3}. \quad (23)$$

*Proof.* Then, we will provide the solution to **Problem 3**. The Lagrangian of **Problem 3** can be written as

$$\begin{aligned} \mathcal{L}_3 = & f\left(\sum_{i=1}^N \mu_i\right) - \xi \sum_{i=1}^N \left(\frac{\mu_i}{q_{ti}} + \frac{\psi}{q_{mi}} + \frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi}\right) \\ & - \alpha_1 \left(\frac{\mu_i}{q_{ti}} p_{ti} - \bar{R}_{ti}\right) - \alpha_2 \left(\frac{\psi}{q_{mi}} p_{mi} - \bar{R}_{mi}\right) \\ & - \alpha_3 \left(\frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi} - \rho_i \mu_i q_{ti}^2 - \rho_i \psi q_{mi}^2\right). \end{aligned}$$

where  $\alpha_1, \alpha_2$  and  $\alpha_3$  are the Lagrange multipliers. The KKT conditions are similar to **Problem 2** except for the following three conditions:

$$\begin{aligned} \alpha_3 &\geq 0, \forall i, \\ \alpha_3 \left(\frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi} - \rho_i \mu_i q_{ti}^2 - \rho_i \psi q_{mi}^2\right) &= 0, \forall i, \\ \frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi} - \rho_i \mu_i q_{ti}^2 - \rho_i \psi q_{mi}^2 &\geq 0, \forall i. \end{aligned}$$

We then analyze the solutions under different cases. Actually, there should be nine cases in this problem, but we only consider two of them to analyze since the other situation can be interpreted similarly.

**Case1:**  $\alpha_1 = \alpha_2 = \alpha_3 = 0, \forall i$ .

In this case, we can have

$$\frac{\partial \mathcal{L}_3}{\partial p_{ti}} = \frac{-(\mu_i \xi (p_{mi} + 2p_{ti}))}{3\rho_i \left(\frac{p_{ti}}{\rho_i}\right)^{\frac{4}{3}}},$$

and let it equal to 0 we can get  $p_{ti} = \frac{-p_{mi}}{2}$ . Obviously, since  $p_{ti}$  and  $p_{mi}$  are non-negative values, we cannot find a  $p_{mi}$  to satisfy the above equation. So this case is invalid.

**Case2:**  $\frac{\mu_i}{q_{ti}} p_{ti} + \frac{\psi}{q_{mi}} p_{mi} - \rho_i \mu_i q_{ti}^2 - \rho_i \psi q_{mi}^2 = 0, \frac{\psi}{q_{ti}} p_{ti} - \bar{R}_{ti} = 0, \alpha_2 = 0, \forall i$ . By solving the above equations, we get (22) and (23). We can verify that the solutions above are incentive compatible and satisfy all the KKT conditions.

Thus **Theorem 4.1** is proved.  $\square$

The optimal solution for  $p_{ti}^*$  in the incomplete information case is the same as the optimal solution in the complete information case, while  $p_{mi}^*$  is different. Since the decision of the MO in the case of incomplete information is not only influenced by the budget of the reward but also required to satisfy the two conditions (20) and (21) in the above definitions. In other words, the decisions in this case are more conservative so the MO would prefer to minimize its cost by reducing the payments to training and mining. We will illustrate the specific differences in the decisions in the two scenarios through experiments in Section 5.

The resource allocation mechanism in the incomplete-information case is presented in Algorithm 2, which is

similar to Algorithm 1, except for the decision process of the MO. In the scenario, the MO should ensure that its utility is optimal and that the utility for each client is non-negative (Lines 2-7). We can see from the pseudocode that the computational cost will increase with  $N$ , so the time complexity of Algorithm 2 is  $\mathcal{O}(N)$ .

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#### Algorithm 2 Resource Allocation Mechanism with Incomplete Information

---

**Require:**  $T, \mu_i, \psi, \rho_i, \eta, \bar{R}_{mi}$

**Ensure:**  $q_{ti}^*, q_{mi}^*, p_{ti}^*, p_{mi}^*$

- 1: The MO calculates  $\hat{p}_{ti}$  and  $\hat{p}_{mi}$  via (22) and (23)
  - 2: **if**  $U_{mo}(\hat{p}_{ti}, \hat{p}_{mi}) \geq U_{mo}(p_{ti}, p_{mi})$  **then**
  - 3:   The MO calculates the expected utility  $\hat{U}_i$  of client  $i$
  - 4:   **if**  $\hat{U}_i \geq 0$  **then**
  - 5:      $p_{ti}^* \leftarrow \hat{p}_{ti}, p_{mi}^* \leftarrow \hat{p}_{mi}$
  - 6:   **end if**
  - 7: **end if**
  - 8: The MO sends  $p_{ti}^*$  and  $p_{mi}^*$  to client  $i$
  - 9: **for**  $i \in \mathcal{N}$  **do**
  - 10:   Calculate  $\hat{q}_{ti}$  and  $\hat{q}_{mi}$
  - 11:   **if**  $U_i(\hat{q}_{ti}, \hat{q}_{mi}) \geq U_i(q_{ti}, q_{mi})$  **then**
  - 12:      $q_{ti}^* \leftarrow \hat{q}_{ti}, q_{mi}^* \leftarrow \hat{q}_{mi}$
  - 13:     Client  $i$  uses  $q_{ti}^*$  to train and  $q_{mi}^*$  to mine
  - 14:   **end if**
  - 15: **end for**
  - 16: **return**  $q_{ti}^*, q_{mi}^*, p_{ti}^*, p_{mi}^*$
- 

We can see that the time complexity of both Algorithm 1 and Algorithm 2 is  $\mathcal{O}(N)$ , which means that the time consumption of solving these two optimization problems increases with the number of clients linearly. Therefore, our proposed algorithms can work efficiently in practice.

## 5 EXPERIMENTAL EVALUATION

In this section, we will conduct numerical experiments to verify and support our designed mechanism. We first clarify the experimental settings and then illustrate the results. We implement the simulations using Matlab 2019b in macOS 11.0.1 running on an Intel i7 processor with 32 GB RAM and 1 TB SSD.

### 5.1 Experimental Setting

In our experiments, we mainly focus on the impacts of four variables (i.e.,  $\mu_i, \psi, p_{ti}$  and  $p_{mi}$ ) on our designed models under complete and incomplete situations. The basic settings for these simulations are slightly different, and we will clarify the different parts of the settings in each experiment. For simplicity of calculation and presentation, we use GHz as the unit of CPU cycles per second and minute as the unit of time. We first set  $\eta = 1500$  and  $\bar{R}_{mi} = 5$ . Since we adopt SV to calculate the total rewards distributed to the individual client and SV is correlated to the value of  $\mu_i$  (see (3) and (4)), we let  $G = \frac{\sum_{i=1}^N \mu_i}{N}$ . By running the algorithm of SV we can get the value of  $\bar{R}_{ti}$  for each client. The settings for other parameters are  $\rho_i = 0.01, \xi = 0.1, g = 10$  and  $T = 15$ . Note that we conducted extensive experiments with other experimental settings, while we found that different values of the parameters would not influence the trends of



the results. So we only present the results of the experiments based on the above settings.

## 5.2 Experimental Results

First, we prove the correctness of the optimal strategies derived from our models. We assume there are 50 clients in total and each client has the same data size, so we set  $\mu_i = 10$ . In our experiments, for clients and the MO, there are four strategy combinations, i.e., both sides choose the optimal strategies, one chooses the random strategies while the other chooses optimal strategies, and both choose the random strategies. For example, we define the strategy combination *Random vs. Optimal* as clients choose the random strategies and the MO chooses the optimal strategy. We compare the utilities of clients and the MO with random strategies and optimal strategies, respectively. The results in Fig. 2 illustrate that clients and the MO can obtain higher utilities than all other strategies when they both choose the optimal strategies, proving the validity of our proposed optimal strategies.

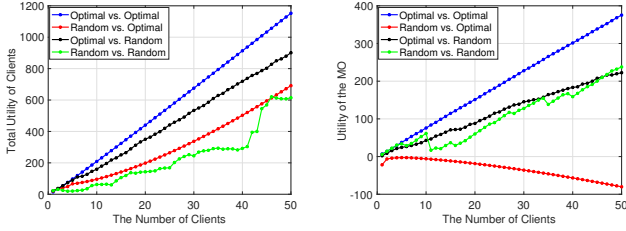


Fig. 2: Utilities changing with strategy pairs.

Then, the experiments will be designed to study the impacts of  $\mu_i$  and  $\psi$  on the utility of clients and the MO under the situations of complete and incomplete information. We set  $\mu_i \in [0, 5]$  and  $\psi \in [0, 5]$ . The simulation results are shown in Fig. 3. We can see that both  $\mu_i$  and  $\psi$  have a significant impact on the utility of the MO. That is because the higher CPU power will shorten the time in each round and improve the performance of the global model. However, for clients, devoting more CPU cycles does not result in more utility due to higher energy consumption.

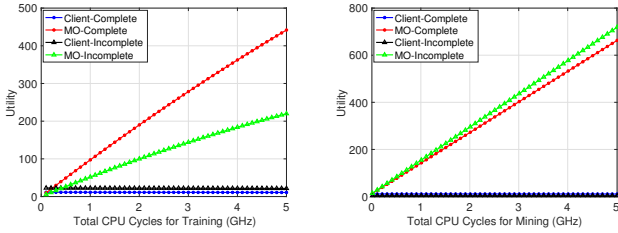


Fig. 3: Utilities of the client and the MO changing with the total CPU cycles for training and mining.

We then study the effect of  $p_{ti}$  and  $p_{mi}$  on the utility of the MO and clients. We set  $p_{ti} \in [0, 10]$  and  $p_{mi} \in [0, 10]$ . The results are shown in Fig. 4. If the unit price of training increases, clients can be stimulated to provide more computing power, which reduces the time cost and improves the model performance so that the MO utility will be improved.

However, the revenue of clients does not grow significantly with the increase of the unit price of training because the cost of energy consumption also rises.  $p_{mi}$  has the same effect on utility for both complete and incomplete information cases, and the results are shown on the right side of Fig. 4. When the unit price of mining increases, the utility of both clients and the MO can be improved. This is because with the increase of  $p_{mi}$ , clients can receive more mining revenue by devoting more  $q_{mi}$ . At the same time, the MO can reduce the time cost and improve its utility by encouraging clients to devote more CPU cycles per second to mining.

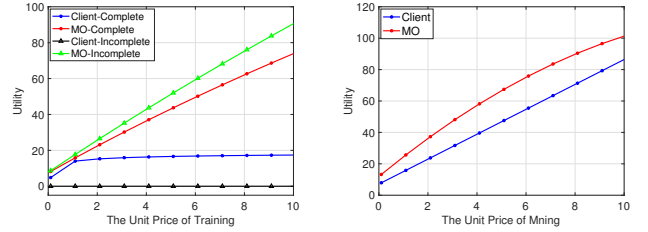


Fig. 4: Utilities of the client and the MO changing with the unit prices of training and mining.

Next, we conduct experiments to analyze the relationship between  $\mu_i$  and the unit price for training and mining. We set  $\mu_i \in [0, 5]$ , and the results are illustrated in Fig. 5. We can see that both the unit price and the number of CPU cycles for training increase with  $\mu_i$ . If  $\mu_i$  increases, more rewards are needed to motivate clients to put more computational resources into training. In general,  $\mu_i$  does not affect  $p_{mi}$  and  $q_{mi}$  a lot, as the benefits of mining are relatively constant and are more influenced by the resource allocation scheme.

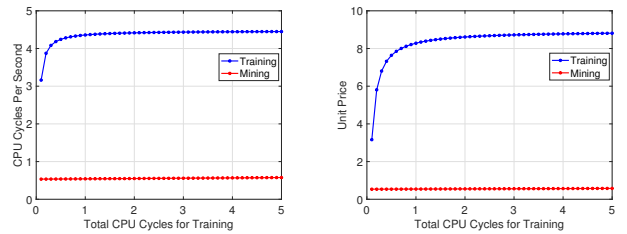


Fig. 5: Impacts of  $\mu_i$  on CPU cycles per second and unit prices for training and mining.

In the end, we explore the influence of  $p_{ti}$  on both  $q_{ti}$  and  $q_{mi}$  to figure out how the decisions of MO influence the decisions of client  $i$ . We set  $\mu_i = 10$  and  $p_{ti} \in [0, 10]$ . In this setting, the simulation results are shown in Fig. 6. We can see that the unit CPU cycles used in local model training have a positive relationship with the unit price of training offered by the MO because more unit rewards for training will incentivize clients to put more computational power into model training. As for CPU cycles per second used in mining, it decreases with the increase of  $p_{ti}$ . This makes sense because if clients are motivated to put more computing power into training, the training time will be reduced and the mining time will be correspondingly increased. In this way, clients do not need to set a high  $q_{mi}$  for mining.

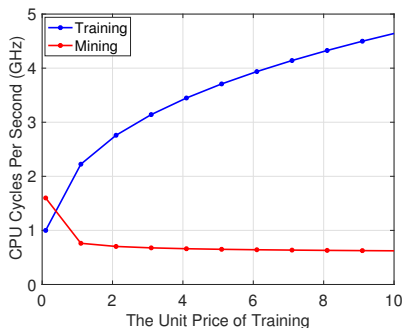


Fig. 6: CPU cycles per second for training and mining changing with  $p_{ti}$ .

## 6 RELATED WORK

Most of the existing studies related to BCFL focus on protecting privacy, achieving decentralization, and improving the performance of model training [33]–[37]. Our paper mainly focuses on resource allocation and incentive mechanism design in BCFL. Thus, we provide the literature review of these two areas in this section.

### 6.1 Resource Allocation in BCFL

As for resource allocation, researchers mainly consider the homogeneous computational power of all clients and make decisions through the reinforcement learning approach.

In [19], the resource allocation problem is resolved for the local devices with the same computational power in BCFL. An upper bound of the global loss function was proposed to evaluate the performance of training; in the meantime, the relationship among update rounds, block generation rate, and learning rate was explored. Although the proposed method can easily control the training and mining time by adjusting the number of updates to allocate resources, it is based on the assumption that all clients have the same amount of computing resources and local data, which is not practical. Hieu *et al.* [20] design a deep reinforcement learning approach to help mobile devices determine the data volume and energy used for training and to assist the system in deciding the block generation rate. Yang *et al.* [21] propose a trustworthy BCFL framework to address the privacy and security issues of FL, where a joint optimization mechanism is designed to allocate communication and computing resources. In [22], a dynamic resource allocation scheme is designed to optimize the process of client selection and model training jointly for the BCFL system, which can efficiently improve the performance of the global model.

According to the above discussion, it can be seen that the studies related to resource allocation in BCFL are insufficient. One of the reasons is that the research regarding BCFL is still in the early stage. Another reason is that there are many types of BCFL structures depending on the role the blockchain plays in FL, making it difficult to have a common framework for resource allocation. In order to assist the MO and the clients of the BCFL system in making the proper decisions, we design the mechanisms based on the two-stage Stackelberg game in this paper. Besides, we

consider allocating resources in the fully coupled BCFL with FL clients working as blockchain nodes.

### 6.2 Incentive Mechanism in BCFL

Some studies about BCFL focus on regulating the behaviors of clients through incentive mechanism design, thus encouraging them to work honestly and efficiently according to the predefined rules.

Toyoda *et al.* [38] propose an economic approach based on the assumption that clients would act rationally, where the repeated competition method was utilized to ensure that clients would follow the protocol. In [24], the authors propose a blockchain-based incentive mechanism to assist the hierarchical federated learning to work in a secure and privacy-preserving way. Kang *et al.* [23] design a data freshness based incentive mechanism to motivate the devices to work efficiently in a BCFL system. Bao *et al.* [39] design an incentive mechanism to attract more data and computational power contributing to the framework of BCFL. In their proposed system, honest clients can gain fairly partitioned rewards while malicious clients will be punished via a timely behavior detection scheme. In [40], an incentive mechanism that integrated reputation and contract theory is proposed to encourage clients to provide high-quality data to train the local models. As for the fairness of reward allocation, Liu *et al.* [41] use Shapley Value (SV) to calculate the contributions of clients of the FL system and then allocate the rewards accordingly. However, this approach is not able to make incentive decisions for training and mining, respectively.

The existing studies about incentive mechanism design in BCFL focus on how to provide incentives for FL through blockchain without considering the incentives for blockchain and FL in a systematical manner. In other words, blockchain and FL are in different phases for BCFL, so they should both have reasonable incentives. In our paper, we design a pricing mechanism for the MO based on the computing power provided by clients, thus providing incentives to the whole BCFL system.

In general, the existing studies have paid little attention to resource allocation for clients in BCFL and assume that clients join the task voluntarily. To address this challenge, we design a resource allocation mechanism for clients, which also offers reward suggestions to the MO so as to motivate clients to participate in BCFL.

## 7 CONCLUSION

This paper studies the resource allocation of clients in BCFL by designing an incentive mechanism. We describe the interactions between clients and the MO as a two-stage Stackelberg game. Within our model, clients with varying computing power can determine the resources to invest in training and mining based on the rewards provided by the MO through maximizing their utilities, while the MO can also obtain the optimal utility. Since the local training related information of clients may not be known to the MO, we further study the game model and optimal solutions in the incomplete information case. Numerous experimental results show that our proposed mechanisms are effective.

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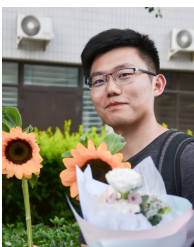
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